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Predrag Cicovacki

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1.

KANT POINTS OUT that he had found in Locke's *Essay concerning Human Understanding* an indication of a general division of judgments into analytic and synthetic. He also notes that in the same work he had found an indication that some of our judgments are (in his terms) synthetic a priori.[1] What Kant says about Locke is peculiar for several reasons. First, Locke himself never used the terms 'analytic', 'synthetic', 'a priori', and 'a posteriori'. Second, when Kant goes on to explain more precisely where he finds an analogy between Locke's and his own view, he refers to passages which do not serve his purpose in the best way.[2]

We can only speculate about Kant's reasons for choosing some passages in Locke rather than others. Yet whatever Kant's reasons are, he gives us a useful conceptual framework which we want to apply here to Locke's view on mathematical knowledge. We hope that in that way we shall be able to show Locke's

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[1] *Prolegomena to Any Future Metaphysics*, trans. and intro. by L. W. Beck (Indianapolis: The Liberal Arts Press, 1950), paragraph 3.

[2] For instance, we expect Kant to say that Locke's division of propositions into trifling and instructive is analogous to his analytic-synthetic distinction. Yet Kant refers to Locke's view on identity or diversity (Kant says "identity or contradiction") as kinds of agreement and disagreement of our ideas like those in analytic judgments. Kant departs significantly from Locke's view, for obviously diverse propositions need not be contradictory. Kant isolates a subset of diverse propositions, those which are contradictory, and joins them with propositions which express identity in order to get a set of all analytic propositions. We might expect that Kant will claim that, according to Locke, our mathematical propositions are synthetic a priori. But Kant remarks that Locke's 'coexistence' propositions are synthetic, and that those of them which express necessary connections are synthetic and a priori (according to his own criteria).

view to be closer to modern points of view, and at the same time show how rich, interesting, and important was his account of mathematical knowledge.

Let us, then, suggest a preliminary dictionary, that is, approximate translations of Locke's terminology into Kant's. According to what Wolfram[3] has called the "Standard Interpretation" (with which, as we shall see, she disagrees), we have

1) instructive (Locke) = synthetic (Kant),
2) trifling (Locke) = analytic (Kant).

Our reasons for the proposed translation are, briefly, the following. Locke claims that trifling propositions are merely verbal and self-evident, and that they do not teach us anything new, that is, they are not instructive. Instructive propositions, on the other hand, are those which enlarge our knowledge; they need not be certain, and when they are, their certainty is not verbal. We do not get instructive propositions by the mere analysis of ("playing with") words contained in our ideas.[4] Hence, what Locke claims about trifling and instructive propositions is similar to what Kant maintains about analytic and synthetic judgments.[5]

2.

The thesis that there is a close resemblance between Locke's view on mathematical propositions and Kant's view on mathematical judgments is not new, but it has not been much elaborated either.[6] The passage which Kant himself does not mention, but which is interpreted by some Locke commentators to demonstrate the analogy between Locke and Kant is the following one:

[W]e can know the Truth and so may be *certain* in Propositions, which affirm something of another, which is a necessary consequence of its precise complex *Idea*, but not contained in it. As that *the external Angle of all Triangles, is bigger than either of the opposite internal Angles;* which relation of the outward Angle, to either of the opposite internal

[3] See Sybil Wolfram's papers, "On the Mistake of Identifying Locke's Trifling-Instructive Distinction with the Analytic-Synthetic Distinction," *The Locke Newsletter* 9 (1978), and "Locke's Trifling-Instructive Distinction—A Reply," *The Locke Newsletter* 11 (1980).

[4] John Locke, *An Essay concerning Human Understanding*, ed. Peter H. Nidditch (Oxford: Clarendon Press, 1975), book 4, chap. 8, par. 3. All subsequent references to the *Essay* will be given in the text of the paper in the following way: (4.8.3).

[5] Cf. *Critique of Pure Reason*, trans. Norman K. Smith (New York: St. Martin's Press, 1929), Introduction to the Second Edition; and *Prolegomena*, par. 2. All subsequent references to the *Critique* will be given in the text, and they will indicate pages in the first edition by the letter 'A' and in the second edition by the letter 'B'.

[6] For instance, James Gibson, *Locke's Theory of Knowledge and Its Historical Relations* (Cambridge: Cambridge University Press, 1917), 317–18. For more references see the first paper by Wolfram cited in note 3.

Angles, making no part of the complex *Idea,* signified by the name Triangle, this is a real Truth, and conveys with it instructive *real Knowledge.*[7] (4.8.8)

In other words, Locke claims that when there is a universal and necessary connection between some complex idea A and some simple or complex idea of the property B, and when the idea of the property B is not contained in A, the proposition which expresses their connection is a real truth, that is, an object of "real Knowledge," not a trifling proposition about the word 'triangle'. The kind of knowledge in question (in the quotation) is demonstrative and if, following Kant, we admit that "necessity and strict universality are thus sure criteria of a priori knowledge" (B16), we shall also admit that this knowledge about the certain and universally valid necessary connection between A and B is an example of a priori knowledge. Thus, from Kant's point of view, this example is obviously an example of a synthetic a priori judgment.

Strangely enough, Kant himself was never aware of that. Two things lead me to such a conclusion. First, the *Critique* (B14) and the *Prolegomena* (par. 2c) leave very little doubt that Kant thinks that no philosopher before him had realized that mathematical propositions are synthetic a priori. Second, the only text in which he mentions Locke's view on mathematics suggests that Kant believes that mathematics is empirical for Locke: "Leibniz wanted to refute the empiricism of Locke. For this purpose examples taken from mathematics were quite well suited to prove that such cognitions reach much further than our empirically acquired concepts could, and therefore to defend the a priori origin of the mathematical cognitions against Locke's attack."[8]

Kant does not explicitly express his own view here, but he neither denies nor disagrees with Leibniz's view. Hence, it is quite possible that he thinks Leibniz is correct, i.e., that Locke really considers mathematical knowledge to be empirical.

Yet the passage quoted from Locke suggests that in spite of the fact that the knowledge which we have of mathematical propositions is instructive, not trifling, mathematical knowledge is, nevertheless, based on necessary connections, and hence is a priori. Thus Locke does not in fact regard mathematical knowledge as empirical.

On the other hand, Sybil Wolfram also denies that, from Locke's point of view, mathematical propositions are synthetic a priori.[9] Wolfram admits that it is not problematic that they are a priori and that they are not trifling; yet she

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[7] All emphases in this and all other quoted passages are Locke's, unless otherwise noted.

[8] Henry E. Allison, ed. and trans., *The Kant-Eberhard Controversy* (Baltimore and London: The Johns Hopkins University Press, 1973), 126–27, note.

[9] Wolfram claims "that far from seeing mathematical propositions as synthetic in any ordinary use of 'synthetic', Locke considered them to be analytic in every common use of 'analytic' " ("On the Mistake," 28.)

claims that they are analytic, not synthetic. Her criticism is summarized in the following statements: " '[S]ynthetic' is normally taken to involve being about the world. . . . And there seems [to be] no possible doubt that someone who describes a set of [mathematical] propositions as 'only of our own ideas' (4.4.6), and not intended to correspond to reality (2.31.3, 4.4.5–6, 8), but demonstrable by deduction from definitions (4.2.1–7, etc.), cannot be maintaining that they are synthetic in any remotely recognizable sense of that much abused term."[10]

Before commenting on the principal substance of Wolfram's statement, I must take a moment to call attention to her use of 'synthetic', claiming that "being about the world" is the only "remotely recognizable sense of that much abused term." Historically, synthetic means what the inventor of the term meant by it, viz., ampliative, adding to our knowledge of a term. To use a contemporary idiom (in which analytic means about the same as a priori, and synthetic means about the same as a posteriori) in explicating a seventeenth-century text is anachronistic. Though it is important to know whether Wolfram is correct in saying that Locke did not believe that mathematical propositions were intended to refer to things in the world, it is also important to separate this question from the question whether his mathematical propositions are synthetic in the historical sense of this word. Kant himself thought that mathematical concepts "are not by themselves knowledge except on the assumption" that there are things in the world which conform to these concepts (B147), but this was a consequence of a long and complex argument, not an analytical consequence of a definition of 'synthetic'. We shall show later that Wolfram is incorrect in denying that Locke holds mathematical knowledge to be synthetic, both in her own conception of syntheticity and in Kant's.

Now let us consider Wolfram's objection that mathematical propositions are analytic because demonstrable by deduction from definition. This assertion, it seems to me, does not have any textual support. Wolfram refers to 4.2.1–7 as her evidence, yet Locke does not use the term 'definition' in the whole chapter to which Wolfram refers, and hence does not say there that mathematical propositions are demonstrable from definitions. Furthermore, it is clear from what Locke says on demonstrative and intuitive knowledge that he does not think so either. Here is one of his examples:

Thus the Mind being willing to know the Agreement or Disagreement in bigness, between the three Angles of a Triangle, and two right ones, cannot by an immediate view and comparing them, do it: Because the three Angles of a Triangle cannot be brought at once, and be compared with any other one, or two Angles; and so of this the Mind has no

[10] Ibid., 46. I do not discuss here all of Wolfram's objections, but only the most relevant ones for our issue. For some other objections see the articles cited above, and for a criticism of the first paper see R. G. Meyers, "Locke, Analyticity and Trifling Propositions," *The Locke Newsletter* 10 (1979): 83–96.

immediate, no intuitive Knowledge. In this Case the Mind is fain to find out some other Angles, to which the three Angles of a Triangle have an Equality; and finding those equal to two right ones, comes to know their Equality to two right ones. (4.2.2)

What Locke claims here is the following. We cannot have intuitive knowledge of the proposition that the three angles of a triangle are equal to two right angles, because we cannot *compare* them *immediately*. We can have only demonstrative or mediate knowledge of that proposition, not as Wolfram argues by deduction from definitions, but by finding some indirect way to *compare* three angles of a triangle with two right angles and in that way to establish that they are equal. The angles which we should compare with each other in order to demonstrate the truth of that mathematical proposition are not given, but we are "fain to find" them (Kant would say: to construct them).

In some other passages where Locke does speak about definitions, he claims that "*a Definition is* nothing else, but *the showing the meaning of one Word by several other non synonymous Terms*" (3.4.6), or that a definition is an explanation of the meaning of the term that stands for an idea (3.3.10). If Locke were to defend the view that mathematical propositions are analytic because demonstrable from definitions (which Wolfram ascribes to him), he would not claim that we have to *compare* the angles in order to establish that they are equal; it would suffice for him to say that we have only to explain and understand the meaning of the word 'triangle'. Thus, Wolfram is wrong in her criticism that mathematical propositions are analytic in the sense of being logical consequences of definitions.

Consider now the objections to the two claims that mathematical propositions are not synthetic because they are "only of our own ideas" and because they are not intended to correspond to reality.

The clue to Locke's understanding of the objects of mathematical propositions is given in his view on complex ideas of modes and relations:

[They] *are* Originals, and *Archetypes*, are not Copies, nor made after the Pattern of any real Experience to which the Mind intends them to be comformable, and exactly to answer. These being such Collections of simple *Ideas*, that the Mind it self puts together, and such Collections, that each of them contains in it precisely all that the Mind intends that it should, they are Archetypes and Essences of modes that may exist; and so are designed only for, and belong only to such Modes, as when they do exist, have an exact conformity with those complex *Ideas*. (2.31.14)

As examples of complex ideas of modes,[11] mathematical ideas are archetypes made by mind, and not copies made on the basis of our experience of

[11] Locke sometimes says that they are examples of relations (see, for instance, 4.1.7); however this "shift" is not so significant since Locke thinks "what has been said of mixed modes is with very little difference applicable also to relations" (3.5.16).

really existing objects, and Wolfram is indeed right in emphasizing that mathematical judgments are "only of our own ideas." Locke distinguishes them quite clearly from empirical propositions which are intended to refer to existing things but do so without necessity (a posteriori). That our mathematical ideas are archetypes made by us without pattern or reference to any existence (3.5.3) means that they are not given in sensation (at least not in the way in which our simple ideas are). What Locke is concerned with here is only the origin of our ideas. He thinks that our knowledge of mathematical propositions is not empirical, that is, that they are known (in Kant's terms) a priori (4.8.8, 2.31.11), and by claiming that our mathematical propositions are "only of our own ideas" Locke does not indicate anything more than that. In other words, the claim that mathematical propositions are "only of our own ideas" has nothing to do with their alleged nonsyntheticity; it simply leaves open the question whether they are analytic or synthetic.

Wolfram's other claim that, according to Locke, mathematical propositions are not intended to correspond to reality, and therefore are not synthetic, is very misleading. It can mean two things:

1) (not more than) that archetypes are not copies of existing things, or
2) that there is no correspondence between archetypes and existing things.

1) does not imply that mathematical ideas are not about the world, that is, that they are not synthetic (in Wolfram's definition of 'synthetic'). Only 2) can establish that. However, it is clear that Locke holds 1) and not 2). What Locke claims in, for instance, 3.5.3, 2.31.11 and 4.4.6 is that mathematical ideas are archetypes and *not copies* of existing things, not that there is no correspondence between mathematical ideas and existing things. For Locke (as well as for Kant) the former claim does not imply the latter. The correspondence between existing things and mathematical ideas does not require that the existing things be the originals and that mathematical ideas be copies of them; the relation may, indeed, be the converse.

Moreover, Locke denies 2). He claims that our ideas of mixed modes and relations are real, and that "there is nothing more required to this kind of *Ideas* to make them *real*, but that they be so framed, that there be a possibility of existing [things] conformable to them" (2.30.4). And when their corresponding objects do exist, they have a conformity with those complex ideas.

Locke's second reply to the Bishop of Worcester shows even more clearly that he denies 2), i.e., the claim that there is no correspondence between archetypes and existing things. Locke there first grants "that those ideas on which mathematical demonstrations proceed, are wholly in the mind," and then claims that "no demonstration whatsoever concerns things as really existing, any further than as they correspond with, and answer those ideas in the mind, which the demonstration proceeds on."[12]

Locke makes a very significant turn (very similar to the one which Kant makes in the so-called Copernican Revolution). In spite of the fact that mathematical ideas are not copies of existing things, Locke claims that there can be, and actually is, an agreement between mathematical ideas (originals) and existing things (copies). Furthermore, for Locke (as well as for Kant) "not intended to correspond to reality" does not imply or mean "not being about the world" and *eo ipso* does not mean or imply not being synthetic (in Wolfram's sense of the word). It seems to me that Locke indicates this turn in the very same paragraph (4.4.6) as that to which Wolfram refers as evidence for her claim that mathematical propositions are not synthetic because not intended to correspond to reality.

> But yet the knowledge [the mathematician] has of any Truths or Properties belonging to a Circle, or any other mathematical Figure, are nevertheless true and certain, even of real Things existing: because real Things are no farther concerned, nor intended to be meant by any such Proposition, than as Things really agree to those *Archetypes* in his Mind. Is it true of the *Idea* of a *Triangle*, that its three Angles are equal to two right ones? It is true also of a *Triangle*, when-ever it really exists. (4.4.6)

Locke does not use the term 'correspondence' here, saying only that mathematical ideas and existing things "agree"; but this is good enough to establish the point in question. There is, or at least might be in principle, an agreement between mathematical ideas and existing things in spite of the fact that the former are "only of our own ideas," and not copies of existing things. Locke maintains that what is true about my archetype of a triangle is also true for any existing triangle (4.4.6). That, of course, is not to say that my judgments of truths about my complex ideas of mode depend on the existence of some real triangle, i.e., on the existence of any physical object having a triangular shape; but if triangles exist outside of the mind, what is true for my idea of a triangle must be also true for all existing triangles (4.4.6). If so, then our ideas of triangles (as well as ideas of any other mathematical object) provide us with

[12] John Locke, *Works*, (London, 1823; reprint, 1963), vol. 4, p. 406. It is interesting to note that the Bishop of Worcester has (mis)understood Locke in the same way Wolfram did. In his response Locke explicitly denies the view which the Bishop and Wolfram ascribe to him: "For you say 'I grant that those ideas on which mathematical demonstrations proceed are wholly in the mind'; this indeed I grant, 'and do not relate to the existence of things'. But these latter words I do not remember that I anywhere say. And I wish you had quoted the place where I grant any such thing. I am sure it is not in the place where it is likeliest to be found: I mean where I examine whether the knowledge we have of mathematical truths be the knowledge of things as really existing" (ibid., p. 405). Locke refers to the *Essay*, 4.4.6, in the quoted passage.

(indirect) knowledge of the external world. Hence, mathematical ideas are synthetic even according to Wolfram's criterion.

3.

We must now return to the question: Why does Locke believe that existing or possible physical figures must correspond to our archetypes?

Like Kant, who claims that the "reason has insight only into that which it produces after a plan of its own" (B xiii), Locke also advocates a version of the theory of maker's knowledge.[13] Except in the case of simple ideas and relations and complex ideas of substances, the mind is active (creative). Complex ideas of mixed modes and relations are not intended to be copies of existing things, nor do they require that there be actual things to which they refer (2.30.4). They are our own products; they are originals or archetypes, as Locke calls them. The fact that they are our own products explains why they are adequate.[14] We do not need to 'collect' properties of our own archetypes by observation. The causes of our archetypes are in us, and as far as we create something ourselves we are able to know its cause and understand its true principles, which are the work of our own minds. On the other hand, we do not have adequate ideas of substances; we do not create their causes nor are we able, according to Locke, to know them. Hence, science of nature is not certain.

This is just a brief summary of Locke's ideas about the theory of maker's knowledge which he repeats time and again.[15] In order to discover some other less well-known distinguishing features of the theory, we have to consider other aspects of his view.

Locke realizes that some of our complex ideas of mixed modes and relations were formed before the combination they stood for ever existed (2.22.2). But our purpose is not just to play with our imagination.[16] We can create as

[13] See also Kant's letter to Plückner, 26 January 1796, where Kant says: "Denn nur das, was wir selbst machen können, verstehen wir aus dem Grunde." For more references and a brief account of Kant's theory of maker's knowledge see Karl Löwith, *Vicos Grundsatz: Verum et Factum Convertuntur* (Heidelberg: Carl Winter-Universitätsverlag, 1968), 25–27. Vico's claim that *verum est ipsum factum* belongs to the same category. Cf. the above-cited book by Löwith, and also James C. Morrison, "Vico's Principle of *Verum* Is *factum* and the Problem of Historicism," *Journal of the History of Ideas* 39, no. 4 (1978): 582–85, and Perez Zagorin, "Vico's Theory of Knowledge: A Critique," *The Philosophical Quarterly* 34, no. 134 (1984): 17–19.

[14] "Those [ideas] I call *adequate* which perfectly represent those archetypes which the mind supposes them taken from, which it intends them to stand for, and to which it refers them" (2.31.1).

[15] See especially the *Essay*, 2.12, 2.22–23, 2.25, 2.30–32, 3.5–6, 3.11, 4.4–8.

[16] "But though these complex ideas, or essences of mixed modes, depend on the mind and are made with great liberty, yet they *are not made at random* and jumbled together without any reason at all" (3.5.7).

many complex ideas of mixed modes and relations as we want without claiming objective truth for them, but we attach names only to some of them. We create names only for those ideas which are useful for our communication (3.5.7–8). There is no reason to form a separate name for, say, just any imaginative idea, or even for the idea of, for instance, a geometrical figure with thirty-seven angles, but it is very useful to have a name for the idea of a figure with three angles (2.22.5; 3.5.7).

The reason for this is purely pragmatical and not epistemic. Yet Locke is not mainly concerned with the pragmatic element; he recognizes some relevant epistemic constraints on our forming complex ideas of mixed modes and relations. For instance, he points out that the first steps from which mathematicians start in making complex ideas are self-evident and undeniable truths (4.12.4, 7) which we know intuitively.[17] In the case of mathematics those self-evident truths are our simple and indefinable ideas of 'unit', 'space', 'point', 'figure', 'duration', 'extension', etc. The human mind "has the Power to repeat, compare, and unite them, even to an almost infinite Variety. . . . But it is not in the Power of the most exalted Wit, or enlarged Understanding, by any quickness or variety of Thought, to *invent or frame one new simple* Idea in the mind" (2.2.2).

This is not to say that, for instance, I cannot think or imagine a point, i.e., I cannot form an idea of a point, if I do not perceive one right now. My forming of such an idea would be based on memories and past experiences (in a very broad sense of that term). But it would clearly be different from inventing such an idea, and this latter is what Locke denies to be possible. Locke takes the impossibility of inventing or making such simple ideas for ourselves to mean that they "*are not fictions* of our Fancies, but the natural and regular productions of Things without us, really operating upon us" (4.4.4). We see, then, that in spite of the fact that our ideas of mixed modes and relations are formed by the free activity of the mind, i.e., without reference to any external things to which they are required to correspond, our simple ideas, which are the necessary basis for complex ideas, give them indirectly a point of connection with experience and reality.

There is yet another constraint here. Locke thinks that our simple ideas must be combined in such a way that the resulting complex ideas are logically consistent. Furthermore, Locke argues that our simple ideas are combined in such a way that there is a possibility of existing things conforming to them (2.30.4). This point is very important for Locke, and the question for him is

[17] See 4.2.9–10, 2.16.13, 4.3.19. See also a very interesting paper by James Gibson, "Locke's Theory of Mathematical Knowledge and of a Possible Science of Ethics," *Mind* 5, no. 17 (1896): 38–40.

not whether our archetypes have objective reference—he has no doubt that many of them do. It is indisputable for Locke that mathematical knowledge is objective real knowledge. But the problem is: How it is possible?

It is quite understandable how copies of things (substances) can provide us with some knowledge about these things. But how can our archetypes provide us with knowledge about the world? Why would something which is self-evidently true about our ideas also be true about things which have a totally different nature? In other words, what guarantees the *fit* (which exists) between our ideas and existing things if our ideas are originals and not copies of those things?

As we have already noted, Locke claims that if there is a triangle in the world, i.e., if there is any object having a triangular shape, the sum of its angles must be equal to the sum of two right angles. Otherwise we would not call the object a triangle, since it does not conform to the archetype. In other words, our archetypes are "patterns," "standards," or "rules" for our choice of names of objects, and unless an experienced object has the properties that we ascribe to a particular archetype, we do not call the experienced object by the name of the archetype. Locke emphasizes that this holds for all kind of archetypes, not solely for the mathematical ones. "[N]othing can be call'd *Gold,*" he says, "but what has a Conformity of Qualities to that abstract complex *Idea,* to which that Name is annexed" (3.3.18).[18]

Hence, we see how our complex ideas of mixed modes can have objective reference. They are *rules* by which we name physical objects in a certain way. "The Mind makes the Patterns, for sorting and naming of Things" (3.5.9).

We face another problem here. If Locke's view on archetypes shows only how we can have a priori knowledge and is indifferent to the distinction between synthetic and analytic knowledge, what, then, would be a Lockean way of distinguishing synthetic from analytic propositions?

The clue can, I believe, be found in Locke's view on essence. He claims that *essentia* in its primary meaning properly means 'being', that is, essence is the very being of one thing whereby it is what it is (3.3.15), and he wants (against the Schoolmen) to preserve this original meaning in his view on real essence. By 'real essence' he means the real inner constitution of anything, the founda-

[18] We are very concerned with the problem of, say, the existence of triangular bodies in the world because we realize that there is an inherent vagueness in the notion of a triangular shape (as well as in any other similar notion). On the other hand, we can notice that Locke does not worry much about the question whether there is any exact conformity between an archetype of a triangle and the triangular shape of a physical body. He is, of course, aware that there are no perfect triangles, circles, etc., in the world, yet he believes that the agreement between our archetypes and physical bodies, which are merely rough approximations of our archetypes, is sufficient for practical purposes.

tion of all its properties, "that particular constitution, which every Thing has within it self, without any relation to any thing without it" (3.6.6). Nominal essence, on the other hand, is always "*the Workmanship of the Understanding*" (3.3.14), and is a collection of properties which, when found in experience, occasions the application of one rather than another name to a substance or mixed mode.

In the case of substances the real essence and the nominal essence need not be the same. The real essence is an inner, invisible constitution (unknown to us) upon which the substance's discoverable properties depend (3.3.15). Locke believes that there is a necessary connection between, e.g., the inner constitution of gold and its malleability, yellowness, etc. It is because of the weakness of our faculties (4.12.10) that we do not know the *modus operandi,* i.e., the way in which the perceptible properties of gold are produced by its inner constitution. Consequently, we are not able to discover this necessary connection (4.12.9). What we are able to discover are only combinations of perceptible properties of gold, and these are objects of probable opinion, not of certain knowledge.[19] All that we know with (a complete) certainty about gold is trifling: the meaning of the word 'gold' (4.6.9) and its species (4.8.13, and also 3.4.7: we rank substances into species according to the complex ideas in us, not according to distinct real essences in them). "And when general Names have any connection with particular Beings, these abstract *Ideas* are the *Medium* that unites them: so that the Essences of Species, as distinguished and denominated by us, neither are, nor can be any thing but those precise abstract *Ideas* we have in our minds. And therefore the supposed real Essences of Substances, if different from our abstract *Ideas,* cannot be the Essences of the Species we rank Things into" (3.3.13).

Let us now return to complex ideas of mixed modes. It is clear that they are archetypes made by us and, hence, that they are significantly different from substances. Yet Locke is willing to talk about real and nominal essence here as well. It is clear that the real essence of mixed modes cannot be corporeal; nevertheless, their real essence is their real inner constitution.[20] Locke's

[19] Locke believes that if we were able to *observe* the internal constitution of substances we would be able to discover and know necessary connections between their properties. But he is wrong here. Our knowing of the *modus operandi* of microscopic corpuscules is neither a necessary nor a sufficient condition for our discovering necessary connections between their properties; it would repeat, on a microscopic level, the embarrassment already experienced on the macroscopic level.

[20] Locke does not tell us much about the real essence of mixed modes, and to speak (as I have done) of the "inner nature" (viz., real essence) of mixed modes might sound quite un-Lockean except when Locke is identifying their real and nominal essence. Yet there is an obvious difference between a mixed mode like Mr. Micawber and a mixed mode like a triangle. Whatever Dickens said about Mr. Micawber is true of Mr. Micawber, since Mr. Micawber is a free creation of

description of the nominal essence of mixed modes seems to be similar to his description of the nominal essence of substances.

But Locke claims that there is "the near relation" between the real and the nominal essence of mixed modes (3.5.10), and even much stronger that they are "the same" (3.5.14). What he means here is the following. First, no discoverable combinations of properties are required, as they are required in the case of substances, to form the nominal essence of a mixed mode. This is why Locke says that "*the Names of mixed Modes always signifie the real Essences of their Species*" (3.5.14), as opposed to the names of substances which signify the combinations of discoverable properties. Second, the real essence of mixed modes is not ontologically different from their nominal essence. Both the real and the nominal essence of mixed modes are abstract ideas; they are made by the mind without patterns, or references to any really existing things. Just what precisely is the ontological status of the real and the nominal essence of mixed modes (as well as of mathematical objects) is something which Locke never clarifies. He tells us that they both "depend on the Mind" (3.5.7). This, of course, can mean different things, but it seems that in the particular context which is relevant for us here Locke does not mean to say more than that complex ideas of mixed modes can be made, abstracted, and have their names given to them, i.e., a species can be constituted, even before any individual of that species ever existed (3.5.5).

When Locke says that the real and the nominal essence of mixed modes are "the same," he does not mean that we have the same kind of knowledge of them. There is an epistemic distinction between the real and the nominal essence of both substances and mixed modes.

In the case of substances we form their nominal essence on the basis of combinations of discoverable perceptible properties. Yet there is no reason to say that the combinations of discoverable properties of one substance and its nominal essence are "the same," at least not from the epistemic point of view. Locke is aware of that problem and he recognizes that sometimes the same

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Dickens's imagination. On the other hand there is something about the simple idea of space which determines the properties of geometrical figures, so that if a spatial figure has three sides, it *must* have exactly three angles. The real essence of the geometrical mixed mode constrains the free creation of geometrical figures. That is, it does not permit us to construct a mixed mode of a quadrilateral triangle, even though trifling propositions following from the nominal essence of such a figure are quite possible. (This distinction underlies Kant's mathematical theory, in which it is essential to distinguish between the *phenomenal essence* and the *logical essence* of a figure. See L. W. Beck, "Analytic and Synthetic Judgments before Kant," in *Essays on Kant and Hume* [New Haven and London: Yale University Press, 1978], 99–100.)

Notice also that the characterization of the real essence of mixed modes in terms of their inner constitution sounds less odd when we know that Locke is willing to treat ideas as (some kind of) real beings. "Ideas may be real beings, though not substances, as motion is a real being, though not a substance" ("Remarks to John Norris' Books," *Works*, vol. 10, p. 256).

sentence is used to express two different propositions, one of which is about a perceptible property of one substance, the other about a part of its nominal essence.

If we, for instance, consider "All gold is malleable" to be a proposition about a discoverable property of one substance (gold), this proposition cannot be known with complete (instructive) certainty (4.6.8–9). This proposition would never be certain because it is a generalization based on our experience of one perceptible property, and as such only probable and never certain, i.e., it would never be known in the strict sense. Contrariwise, argues Locke, "it is a very certain Proposition, if *Malleableness* be a part of the complex *Idea* the word *Gold* stands for. But then here is nothing affirmed of *Gold* [as substance], but that that Sound stands for an *Idea* in which *Malleableness* is contained" (4.6.9). However, the certainty which we have in the latter case is merely verbal, not instructive.

Something similar holds for the relation between the real and the nominal essence of mixed modes. Locke says that "a Figure including a Space between three Lines, is the real, as well as nominal *Essence* of a Triangle; it being not only the abstract *Idea* to which the general Name is annexed, but the very *Essentia* or Being, of the thing it self, that Foundation from which all its Properties flow, and to which they are all inseparably annexed" (3.3.18). His idea is that although the same expression, "a Figure including a Space between three Lines," can be used to describe both the real and the nominal essence of a triangle, they are nevertheless two different aspects of the same mixed mode. One is the real inner constitution of a triangle, "the Foundation from which all its Properties flow, and to which they are all inseparably annexed," while the other is the abstract idea, the medium which connects a particular figure and its general name (3.3.13).

Knowledge which we have about these two different aspects is also different, and this is what is important for us in this context. As we have already seen, Locke believes that if we were able to know the real essence of substances, we would also be able to know synthetically necessary connections between those real essences and their discoverable properties, as well as the necessary relations between the properties. In the case of complex ideas of mixed modes we do know the real essence (because we create it) and consequently are able to have synthetic knowledge of the mixed modes. For instance, the property of all triangles of having an external angle bigger than either of the opposite internal angles is not contained in the nominal essence of triangle (4.4.8); on the contrary, this is a property which follows with necessity from its inner constitution. (We have seen above that it does not follow from the definition or nominal essence of the triangle.) Hence, our propositions about properties of complex ideas which depend upon our knowledge of the real essence of something are known, if at all, *synthetically*, even though we made the complex idea.

On the other hand, in order to know that gold is malleable, we do not have to know what is the real essence of gold. That is to say, when our propositions about properties of complex ideas do not depend on the real essence of something, but rather depend on the nominal essence of it, they are known, if at all, only *analytically*, and the propositions are only trifling.

Locke's theory of maker's knowledge explains how a priori knowledge is possible; it is possible because we create it. The theory does not exclude the possibility that there is synthetic, as well as analytic, a priori knowledge, but it is unable to determine whether any particular knowledge is synthetic or analytic. Locke's view on real and nominal essence, however, provides us with a criterion for judging the syntheticity or analyticity of propositions known a priori. When we know a priori the real essence of a mixed mode, we know it synthetically, because there was not an antecedent concept given for analysis. When we know the nominal essence of a mixed mode, the consequences we draw from it are analytical. Mathematics deals with the real essence of mixed modes and, hence we see why, according to Locke, mathematical knowledge is both a priori and synthetic.

University of Rochester